HESSIAN BARRIER ALGORITHMS FOR LINEARLY CONSTRAINED OPTIMIZATION PROBLEMS

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We develop an interior-point method for conic constrained optimization problems (possibly nonconvex) of the form

$$\min f(x) \quad \text{s.t.: } Ax = b, x \in \bar{C}.$$

$\bar{C}$ is a closed convex cone and A is an $m \times n$ matrix of rank m. Denote the image space of A by $\mathcal{A}$, and the feasible set $\mathcal{X} = \mathcal{A} \cap \bar{C}$. We are given a matrix-valued function H which is adapted to the geometry of C. We use this function to define a variable metric given by $\langle u, v \rangle_{H(x)} := u^\top H(x)v$. The algorithm we are considering is the recursive scheme

$$x^{k+1} = x^k + \alpha^k P_{x^k} H(x^k)^{-1} \nabla f(x^k), \tag{HBA}$$

where P_x is a projection matrix on the null space of the polyhedral subdomain, and H is a matrix-valued function, defining an adapted Riemannian metric for the conic set $\bar{C}$. Hence, the method can be understood from the point of view of splitting techniques where two separate projections are performed to obtain a feasible point in the intersection $\mathcal{A} \cap \bar{C}$, without actually computing a full projection. The method, which we call the *Hessian-Barrier algorithm* combines a forward Euler discretization of Riemmanian-Hessian Gradient flows [1], with ideas from Trust-Region methods, recently proposed in [6], and generalizes Affine scaling schemes, as well as interior point methods based on the replicator dynamics [4]. We investigate viability, stability, convergence and complexity of the algorithm, and potentials for acceleration and compare its computational advantage to state-of-the-art first-order methods. In particular, we describe new first-order techniques based on recent development of generalized self-concordant functions [6]. Our main result is that, modulo a non-degeneracy condition, the algorithm converges to the problem's set of critical points; hence, in the convex case, the algorithm converges globally to the problem's minimum set. In the case of linearly constrained quadratic programs (not necessarily convex), we also show that the method's convergence rate is $O(1/k^\rho)$ for some $\rho \in (0, 1]$ that depends only on the choice of metric (i.e. not on the problem's primitives). These theoretical results are validated by numerical experiments in standard non-convex test functions and large-scale Traffic Assignment problems.

We will also report on recent advances on (HBA) when applied to non-convex, non-smooth problems, with applications to non-convex statistical estimation problems and neural networks.

This talk is based on joint recent work together with Immanuel M. Bomze, Panayotis Mertikopoulos and Werner Schachinger [5].

References

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